Comment on ''Action at a distance as a full-value solution of Maxwell equations: The basis and application of the separated-potentials method''

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Chubykalo and Smirnov-Rueda [Phys. Rev. E **53**, 5373 (1996); **55**, 3793(E) (1997)], introduced the ''electrodynamics dualism concept'' with the simultaneous coexistence of instantaneous and short range interactions. They argued the incompleteness of the existing set of solutions of Maxwell equations and thus of the inadequacy of Liénard-Wiechert potentials to describe the whole electromagnetic field. In contrast we have shown that the usual Liénard-Wiechert retarded time solution is a full-value solution and complete solution of Maxwell equations. Using Villecco's recent work [Phys. Rev. E **48**, 4008 (1993)] we have additionally justified our results concluding that there is always one complete solution that can be presented in different but equivalent forms: retarded-time representations, advanced-time representations, the linear combination of retarded- and advanced-time solutions, or as an action at a distance, i.e., the present-time formulation.
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Recently Chubykalo and Smirnov-Rueda [1] stated the inadequacy of the Liénard-Wiechert (LW) potentials to describe the whole electromagnetic field and the incompleteness of the existing set of solutions of Maxwell equations (ME). They have started with the LW expression for the electric field created by an arbitrarily moving charge, Eq. (1) in [1], and specified it for the case of a charge q moving in a laboratory reference system with a constant acceleration a along the positive direction of the x axis, their Eq. (5). In an erratum [1] they corrected their result, Eq. (7), that Eq. (5) does not satisfy Eq. (2), and showed (as we also did) that the right hand side of their Eq. (7) is zero. They introduced in [1] the ''electrodynamics dualism concept'' according to which [1]: ''there is a *simultaneous* and *independent* coexistence of Newton instantaneous long-range (NILI) and Faraday-Maxwell short-range interactions (FMSI), which cannot be reduced to each other.'' Instead of the usually obtained Eq. (1) they have found by means of the mathematical method (so-called separated-potential method) the expression (27) in which the first term on the right hand side is responsible for the instantaneous aspect of the electromagnetic interaction. This expression is actually the same as a present-time or instantaneous action-at-a-distance representation of the electric field of an uniformly moving charge. The second term is responsible for explicit time-dependent phenomena. Thus, Maxwell's equations have been decomposed in [1] into *two independent sets of equations* (for potentials these are equations (21)–(24) in [1]). The general solution for potentials is given by Eqs. (19)–(20), and for fields by Eqs. (25)–(27) in [1]. The rest of the paper [1] completely relies on the mentioned separated-potential method. At the end of the erratum [1] they stated: ''These errata do not influence the results and conclusions of the paper.''

However, they have found in the erratum [1] (and we also) that the LW retarded solution for $E_x(x,y,z,t)$, Eq. (5) in [1], is consistent with the wave Eq. (2) in [1] and that the right hand side of Eq. (7) in [1] is zero. This means that the LW potentials, as the solutions of the complete set of MEs, are adequate for describing properties of electromagnetic field along any direction including the direction of an arbitrarily moving charge. Thus the retarded LW potentials are the solutions of the D'Alembert equation for any motion of the charge. There is no need to introduce the separate-potential method of [1].

We now make some additional remarks in connection with the LW solutions. The electromagnetic field generated by an arbitrarily moving point charge in free space, Eq. (1) in [1], may be obtained in different ways. One way is by means of a fields-only approach, i.e., without using intermediate vector and scalar potentials, and thus dispensing with the need for gauge conditions. Such methods are, for example, the Fourier transform method, e.g., [2] and [3], the method of Green's functions, e.g., [4], or by means of a time-dependent generalization of Helmholtz's theorem [5]. In any case, dealing with a fields-only approach, one can say that the Liénard-Wiechert fields are the solutions of the inhomogeneous wave equation obtained from MEs,

$$\Delta\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=\frac{\nabla\rho}{\varepsilon_0}+\mu_0\frac{\partial\mathbf{j}}{\partial t}. \qquad (1)$$

Thus, for example, the expression for $E_x(x,y,z,t)$, Eq. (5) in [1], is the solution of the homogeneous wave equation (2) in [1]. But it is well known that if the **E** and **B** fields satisfy the Maxwell equations then they must also be a solution to the wave equations, (1) for the **E** field. The converse is not generally true since there are solutions to the wave equation, e.g., Eq. (1), that are not solutions to the Maxwell equations; see e.g., [6]. Since it is proved that Eq. (5) in [1] is the

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solution to the wave equation (2) in [1], we still have to prove that it is the solution to the Maxwell equations. To prove it, according to [6] we have to show that Eq. (5) in [1] satisfies the Maxwell equations at a particular moment of time $t=0$. Then Eq. (5) in [1] will satisfy the MEs at any time t.

We quote only the result for the Gauss law. Thus one has to prove that

$$\nabla \mathbf{E}(x,0,0,t)|_{t=0}=\mathbf{0}, \tag{2}$$

where $\mathbf{E}$ is given by Eq. (5) in [1]. Then we find

$$\left.\frac{\partial E_x(x,0,0,t)}{\partial x}\right|_{t=0} = -\frac{2Kq}{(x-x_0)^3[1+(a/c^2)(x-x_0)]^2} + \frac{2Kqa^2}{c^4(x-x_0)[1+(a/c^2)(x-x_0)]^3},$$

$$\left.\frac{\partial E_y(x,0,0,t)}{\partial y}\right|_{t=0} = -\frac{1}{2}\left.\frac{\partial E_x}{\partial x}\right|_{t=0}, \tag{3}$$

$$\left.\frac{\partial E_z(x,0,0,t)}{\partial z}\right|_{t=0} = \left.\frac{\partial E_y}{\partial y}\right|_{t=0},$$

and thus Eq. (2) holds; $K=1/4\pi\varepsilon_0$ and y, y_0, z, z_0, t approach zero after differentiation. In a similar manner one can see that Eq. (5) in [1] satisfies all Maxwell's equations at $t=0$, which means that this solution of the inhomogeneous wave equation is at the same time the solution of *the complete set of the MEs at any time t*. Of course the same holds for Eq. (1) in [1]. This discussion reveals that independently of how the solution, Eq. (1) in [1], is obtained it is the complete and full-value solution of the complete set of MEs.

The next remark refers to the decomposition of the total electric field in terms of its transverse and longitudinal components, and to the fact that the longitudinal component propagates instantaneously. Namely, if one writes $\mathbf{E}=\mathbf{E}_T+\mathbf{E}_L$, where $\nabla\mathbf{E}_T\equiv\mathbf{0}$ and $\boldsymbol{\nabla}\times\mathbf{E}_L\equiv\mathbf{0}$, then from the Maxwell equation $\nabla\mathbf{E}_L(\mathbf{r},t)=\rho(\mathbf{r},t)/\varepsilon_0$ it follows that any changes in $\rho(\mathbf{r},t)$ are manifested instantaneously throughout space in $\mathbf{E}_L(\mathbf{r},t)$. On the other hand, *the total electric field*, i.e., Liénard-Wiechert field, Eq. (1) in [1], *is propagated in a retarded fashion*. The way in which the instantaneous effects from $\mathbf{E}_L$ are removed in the total field is explained in [2]. Namely, it was shown in [2] by the Fourier transform method that the space-time transverse electric field contains a term that exactly cancels the instantaneous longitudinal electric field. The total electric field, Eq. (1) in [1], contains effects due to both the velocity and the acceleration of the particle, but all taken at the retarded time t_0, which means that only Faraday-Maxwell short range interaction remains in the total $\mathbf{E}$. But, it has to be noted here that when the solution of MEs is written in the retarded form, as it is Eq. (1) in [1], then the total $\mathbf{E}$ field is the sum of $\mathbf{E}_T$, $\mathbf{E}_L$, and an additional term. This follows from the generalized Helmholtz theorem (see [5]), which yields

$$\mathbf{E}=-\boldsymbol{\nabla}\int\frac{[\boldsymbol{\nabla}'\mathbf{E}]}{4\pi r}\,d\nu'+\boldsymbol{\nabla}\times\int\frac{[\boldsymbol{\nabla}'\times\mathbf{E}]}{4\pi r}\,d\nu' + \frac{1}{c^2}\frac{\partial}{\partial t}\int\frac{[\partial\mathbf{E}/\partial t]}{4\pi r}\,d\nu'. \tag{4}$$

Equation (4) is Eq. (14) in [5], and the notation is that one from [5].

The first term is $-\boldsymbol{\nabla}\varphi$, and it is $\mathbf{E}_L$. The second term is $\mathbf{E}_T$, and the second and the third terms taken together come from $-\partial\mathbf{A}/\partial t$, since $\mathbf{E}$ always can be written as $\mathbf{E}=-\boldsymbol{\nabla}\varphi-\partial\mathbf{A}/\partial t$. The simple decomposition of $\mathbf{E}$ as $\mathbf{E}=\mathbf{E}_T+\mathbf{E}_L$ is valid when $\mathbf{E}$ is written in the present time formulation.

The preceding discussion was aimed at showing that the Liénard-Wiechert retarded solutions of Maxwell's equations are complete solutions and thus sufficient to describe the whole electromagnetic phenomenon. However, recently [7] Villecco developed an exact theory for the present-time or action-at-a-distance formulation of the classical electromagnetism, (see also [8] for the present-time formulation of arbitrary time-independent charge distributions moving with constant velocity). In [7] the LW potentials and electric and magnetic fields are developed in an instantaneous action-at-a-distance format in terms of Lagrange series expansion, and it has been shown that such present-time expressions are completely equivalent to the classical retarded-time expressions. The general procedure has been given in [7] for translating any classical retarded-time expression into a present-time formulation. Furthermore in [7] the same procedure with Lagrange series expansion has been applied to advanced time potentials and fields for translating them into the present-time expressions. Villecco's work explicitly shows that there are no two independent solutions for the complete set of Maxwell equations as proposed in [1], and thus that there is no need for the ''electrodynamics dualism concept.'' Namely, it is argued in [1] that, e.g., the solutions of MEs for the electric field, Eq. (27) in [1], consists of two independent parts of which one is a free electric field $\mathbf{E}^*$ that can be transferred locally, i.e., that propagates in a retarded fashion, and the other part is the action-at-a-distance part $\mathbf{E}_0$, [1], ''linked exclusively to charges (currents) and responsible for interparticle interaction, which *cannot be transferred locally* in space.'' However from [7] it immediately follows that there is only one solution of the complete set of MEs, but this solution can be presented in different equivalent forms; in retarded-time representation, advanced-time representation, or in a completely equivalent, present-time formulation. (Particularly interesting and important is Villeco's discussion about the mixed potentials and the causality parameter.) Since the procedure from [7] for translating the retarded-time expressions into the present-time formulation is quite general, one can translate the retarded-time expression (5) from [1] for $E_x(x,y,z,t)$ of a uniformly accelerated particle into its present-time format, and then substitute it into the wave equation and MEs. This will be presented elsewhere.

In conclusion, we see that in our opinion it is not justified to introduce the separate potential method and dualism concept [1], and to derive from them some conclusions about the changes of the physical meanings of different quantities of classical electrodynamics.

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